SYDNEY GIRLS HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION

Mathematics 2012

General Instructions

- Reading Time- 5 minutes
- Working Time - 3 hours
- Write using a blue or black pen
- Board approved calculators may be used
- A Standard Integrals Sheet is provided at the back of this paper which may be detached and used throughout the paper.

Name:...................................

Teacher:.................................

Total Marks 100

Section I

10 marks

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II

90 marks

- Attempt questions 11 – 16
- Answer on the blank paper provide.
- Start a new sheet for each question.
- Allow about 2 hours & 45 minutes for this section

Question one (1mark)

Simplify $5\sqrt{3}+\sqrt{20}-2\sqrt{12}+\sqrt{45}$

a) $\sqrt{5}-\sqrt{3}$ b) $\sqrt{5}+\sqrt{3}$ c) $5\sqrt{5}+9\sqrt{3}$ d) $5\sqrt{5}+\sqrt{3}$

Question two (1mark)

Find the length of the arc which subtends angle of 15° at the centre of a circle of radius 0.1m.(answer correct to 3 decimal places)

a) 1.500 b) 0.262 c) 0.026 d) 0.008

Question three (1mark)

Solve $|2x-1|=3x$

a) $x=-1$ b) $x=-1$ or $x=\frac{1}{5}$ c) $x=\frac{1}{5}$ d) $x=1$

Question four (1mark)

If $\int_0^a (4-2x)\,dx=4$, find the value of a.

a) $a=-2$ b) $a=0$ c) $a=4$ d) $a=2$

Question five (1mark)

The derivative of the function $y=2x\cos(e^{1-5x})$ is :

a) $y'=-10x\cos(e^{1-5x})$

b) $y'=2\cos(e^{1-5x})+10x\sin(e^{1-5x})(e^{1-5x})$

c) $y'=-10x\sin(e^{1-5x})$

d) $y'=2\cos(e^{1-5x})-10x\sin(e^{1-5x})$

Question six (1mark)

If $\sqrt{7}+\sqrt{28}+\sqrt{63}\dots\dots\dots+p=300\sqrt{7}$. How many terms are there in the series?

a) 24 b) 300 c) 298 d) 25

Question seven (1mark)

Given that the curve $y=ax^2-8x-8$ has a stationary point at $x=2$, find the value of a.

a) $a=\frac{1}{2}$ b) $a=2$ c) $a=6$ d) $a=-2$

Question eight (1mark)

The solution to this equation $\frac{3x-2}{4}-\frac{4-x}{3}=-4$ is:

a) $x=2$ b) $x=5\frac{3}{5}$ c) $x=-2$ d) $x=-5\frac{1}{5}$

Question nine (1mark)

Find the values of m for which $24+2m-m^2\leq 0$

a) $m\leq -4$ or $m\geq 6$ b) $m\leq -6$ or $m\geq 4$ c) $-4\leq m\leq 6$ d) $-6\leq m\leq 4$

Question ten (1mark)

In a game that Batman invented, two ordinary dice are thrown repeatedly until the sum of the two numbers shown is either 7 or 9. If the sum is 9 you win . If the sum is 7 you lose. If the sum is any other number , you continue to throw until it is 7 or 9. The probability that a second throw required is :

a) $\frac{13}{18}$ b) $\frac{1}{9}$ c) $\frac{5}{18}$ d) $\frac{1}{54}$

Question eleven (15 marks)

a) Factorise $4x^2-8x-5$ (2)

b) Solve $3x^3-1=2x.3x^2$ (1)

c) Find the domain and the range of :

i) $f(x)=\sqrt{3-x^2}$ (2)

ii) State whether $f(x)$ is odd or even, giving reasons. (2)

d) Integrate the following:

i) $\int\left(3x^2+\frac{1}{x}\right)^2 dx$ (2)

ii) $\int 4\sin(2x-1)dx$ (1)

e) Given $\log_a 2=x$, find $\log_a(2a)^3$ in terms of x. (2)

f) Find the primitive function of $\frac{3x}{x^2+1}$. (1)

g) Solve $\sin\theta=\sqrt{3}\cos\theta$ for $0\le\theta\le 2\pi$ (2)

Question Twelve (15 marks)

a) Differentiate

i) $y = \dfrac{3x}{7\cos x}$ (2)

ii) $y = 4x^2 \ln(2-x)$ (2)

b) $A(2,-2)$, $B(-2,3)$ and $C(0,2)$ are the vertices of a triangle ABC. Plot the points to form triangle ABC

i) Find the equation of the line AC in general form (2)

ii) Calculate the perpendicular distance of B from the side AC (2)

iii) Find the coordinates of D such that $ABCD$ is a parallelogram (2)

c) Prove $\dfrac{\cos\theta}{1-\tan\theta} + \dfrac{\sin\theta}{1-\cot\theta} = \sin\theta + \cos\theta$ (3)

d) Differentiate $y = \ln\left(\dfrac{2x-3}{x^2+6}\right)$ (2)

Question thirteen **(15 marks)**

a) Consider the function $f(x) = 1 - 3x + x^3$ in the domain $-2 \le x \le 3$

i) Find the stationary points and determine their nature. (3)

ii) Find the point of inflexion. (2)

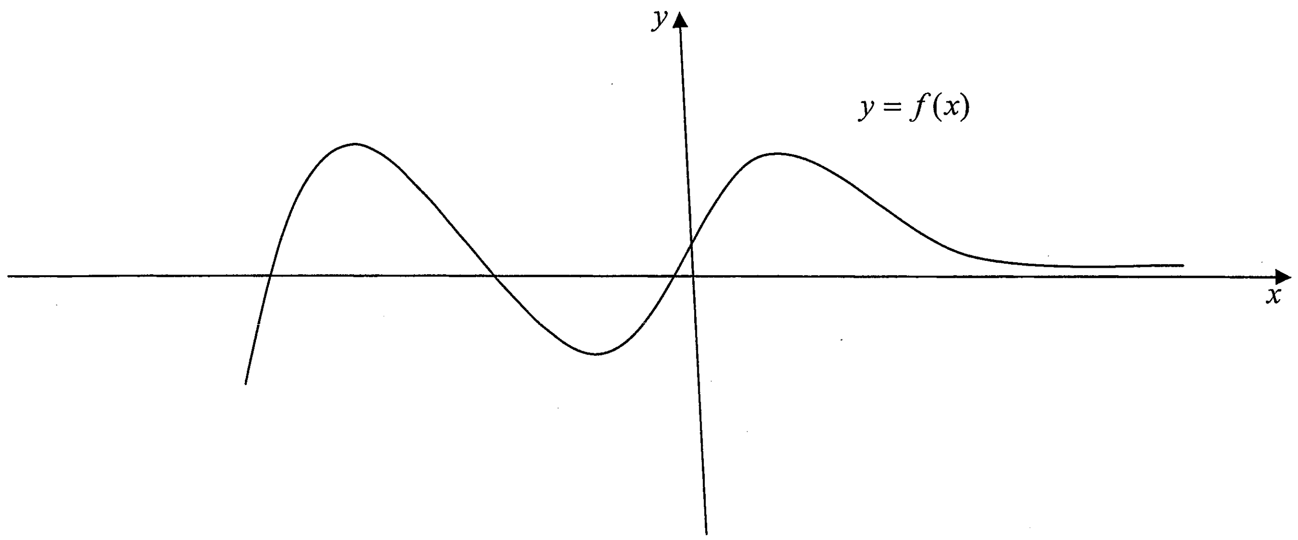
iii) Draw a sketch of the curve $y = f(x)$ in the domain $-2 \le x \le 3$, clearly showing all important features. (2)

iv) What is the maximum value of the function in the given domain? (1)

b) $\int 1 + \sec^2 \pi x \, dx$ (1)

c) The line $y = 3x - p + 2$ is tangent to the parabola $y = x^2 + 1$. Find the value of p. (2)

d) The diagram shows the graph of the function $y = f(x)$, copy the diagram on your answer sheet, and draw the graph of $f'(x)$. (2)



e) Solve the equation $5^{2x} - 4.5^x - 5 = 0$. (2)

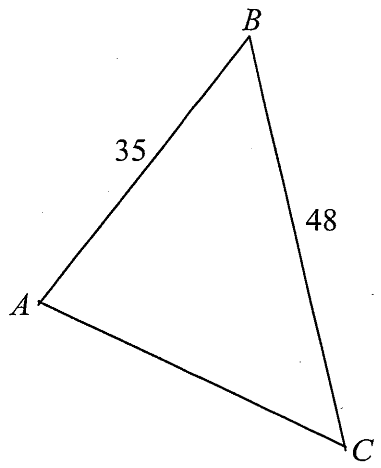
Question Fourteen (15 marks)

a) Consider the curve $y = 3\cos 2x$ in the domain $-\pi \le x \le \pi$

i) State the amplitude and the period of the curve (2)

ii) Sketch the curve in the given domain (1)

b) The bearing of B from A is $036^\circ T$ and the bearing of C from B is $156^\circ T$.



Copy the diagram on to your answer sheet.

i) Find the value of $\angle ABC$. (2)

ii) Find the distance AC. (2)

iii) Find the bearing of A from C. (2)

c) The equation of a parabola is given by $2y = x^2 - 4x + 6$. Find

i) the coordinates of the vertex (2)

ii) the coordinates of the focus (1)

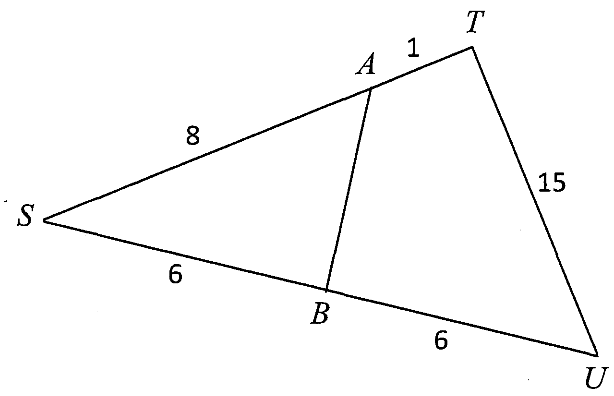
iii) the equation of the directrix. (1)

d) Find the equation of the tangent to the curve $y = 2xe^x$ at the point $(1, e)$. (2)

Question Fifteen (15 marks)

a) Solve $\log_e(2x+2)+\log_e x-\log_e 12=0$ (3)

b)



(Figure not to scale)

i) Prove that ΔSAB is similar to ΔSUT. (2)

ii) Hence find the length of AB (2)

c) Use the Simpson's rule with 4 subintervals to find an approximation for the area of the following figure. All measurements are in metres. (2)

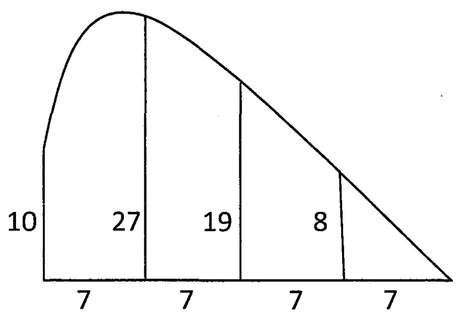


Figure not to scale

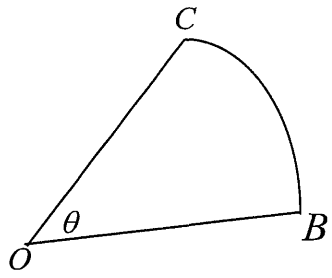
d) Find the exact area bounded by the curve $y=\log_e x$, the line $x=8$, and the axis. (3)

e) For what values of k does the equation $x^2+(k+2)x+4=0$, have distinct real roots. (3)

Question sixteen (15 marks)

a) Find the volume of the solid formed when the area bounded by the curve $y=5-x^2$, for $x\geq 0$, the y-axis and the line $y=1$ is rotated about the x-axis. (3)

b)



The diagram above shows a sector of a circle with centre O and radius r cm.

The arc BC subtends an angle θ radians at O and the area of the sector is 8 cm^2.

i) Find an expression for r in terms of θ (1)

ii) Show that the perimeter of the sector is given by $P=\frac{8}{\sqrt{\theta}}+4\sqrt{\theta}$ (2)

iii) If $0\leq\theta\leq\pi$, find the value of θ for a minimum perimeter. (3)

c) Spiderman worked out that he could save \$80000 in 5 years by depositing his salary of $\$M$ at the beginning of each month into a savings account and withdrawing \$1800 at the end of each month for living expenses. The savings account paid interest at the rate of 6% p.a compounding monthly. Let A_n represent the balance in his savings account at the end of each month.

i) show that at the end of the second month the balance in his savings account, immediately after making his \$1800 withdrawal would be given by : $A_2=(1.005^2+1.005)M-1800(1.005+1)$ (2)

ii) Hence calculate his salary. (2)

iii) How many years will it take him to save \$120000, if he has the same salary and monthly expenses? (2)

THE END

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Multiple Choice - Trial -

1. $5\sqrt{3} + \sqrt{20} - 2\sqrt{12} + \sqrt{45}$

$5\sqrt{3} + 2\sqrt{5} - 4\sqrt{3} + 3\sqrt{5}$

$\sqrt{3} + 5\sqrt{5}$

(D)

2. $l = r\theta$ $\pi = 180°$

$1° = \frac{\pi}{180}$

$l = 0.1 \times 15 \times \frac{\pi}{180}$

$l = 0.026$

(C)

3. $|2x-1| = 3x$

$2x-1 = 3x$ or $2x-1 = -3x$

$-1 = x$ $-1 = -5x$

$x = \frac{1}{5}$

check solutions

$x = \frac{1}{5}$ only solution (C)

4. $\int_0^a (4-2x)\,dx = 4$

$\left[4x - \frac{2x^2}{2}\right]_0^a = 4$

$4a - a^2 = 4$

$a^2 - 4a + 4 = 0$

$(a-2)(a-2) = 0$

$\therefore a = 2$ (D)

5. $y = 2x.\cos(e^{1-5x})$ (u = $2x$, v = $\cos(e^{1-5x})$)

$\frac{dy}{dx} = \frac{u\,dv}{dx} + \frac{v\,du}{dx}$

$= 2x.-\sin(e^{1-5x}) \times -5 \times e^{1-5x} + 2\cos e^{1-5x}$

$= 2\cos e^{1-5x} + 10x\sin(e^{1-5x}).e^{1-5x}$

(B)

6. $\sqrt{7} + \sqrt{28} + \sqrt{63} + \ldots + p = 300\sqrt{7}$

$\sqrt{7} + 2\sqrt{7} + 3\sqrt{7} + \ldots + p = 300\sqrt{7}$

$a = \sqrt{7}$

$d = \sqrt{7}$

$S_n = 300\sqrt{7}$

$\therefore S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$300\sqrt{7} = \frac{n}{2}\left[2\sqrt{7} + (n-1)\sqrt{7}\right]$

$600\sqrt{7} = n\left[2\sqrt{7} + \sqrt{7}n - \sqrt{7}\right]$

$600\sqrt{7} = \sqrt{7}n + \sqrt{7}n^2$

$n^2 + n - 600 = 0$

$(n+25)(n-24) = 0$

$n = -25$ or $n = 24$ (A)

$\therefore$ only solution $n = 24$

7. $y = ax^2 - 8x - 8$

$\frac{dy}{dx} = 2ax - 8$

at $x = 2$, $\frac{dy}{dx} = 0$

$4a - 8 = 0$

$4a = 8$

$a = 2$

(B)

8. $\frac{3x-2}{4} - \frac{4-x}{3} = -4$

$3(3x-2) - 4(4-x) = -4(12)$ (C)

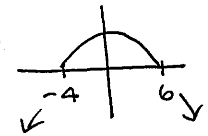
$9x - 6 - 16 + 4x = -48$

$13x = -26$

$x = -2$

9. $24 + 2m - m^2 \leq 0$ (A)

$(6-m)(4+m) \leq 0$



$\therefore m \leq -4$ or $m \geq 6$

10.

1,1	2,1	3,1	4,1	5,1	6,1
1,2	2,2	3,2	4,2	5,2	6,2
1,3	2,3	3,3	4,3	5,3	6,3
1,4	2,4	3,4	4,4	5,4	6,4
1,5	2,5	3,5	4,5	5,5	6,5
1,6	2,6	3,6	4,6		

$P(7) = \frac{6}{36}$ (A)

$P(9) = \frac{4}{36}$

$P(\text{other}) = \frac{26}{36} = \frac{13}{18}$ for a second throw.

Q 11

a) $4x^2 - 8x - 5$ — P −20, S −8

2 $= (2x-5)(2x+1)$

b) $3x^3 - 1 = 2x \cdot 3x^2$

1 $3x^3 - 1 = 6x^3$

$-3x^3 - 1 = 0$

$-3x^3 = 1$

$x^3 = \frac{-1}{3}$

$x = -\sqrt[3]{\frac{1}{3}}$

$= -\frac{1}{\sqrt[3]{3}}$

c) i) D: $-\sqrt{3} \le x \le \sqrt{3}$ 1

2 R: $0 \le y \le \sqrt{3}$ 1

ii) even 1 $f(x) = \sqrt{3-x^2}$

2 $f(x) = f(-x)$, 1 $f(-x) = \sqrt{3-(-x)^2}$

$= \sqrt{3-x^2}$

d) i) $\int \left(3x^2 + \frac{1}{x}\right)^2 dx$

2 $= \int \left(9x^4 + 6x + \frac{1}{x^2}\right) dx$ 1

$= \frac{9x^5}{5} + 3x^2 - \frac{1}{x} + C$ 1

ii) $\int 4 \sin(2x-1)\, dx$

1 $= -4 \times \frac{1}{2} \cos(2x-1) + C$

$= -2\cos(2x-1) + C$ 1

e) $\log_a 2 = x$,

2 $\log_a (2a)^3$

$= 3 \log_a 2a$ 1

$= 3\left[\log_a 2 + \log_a a\right]$

$= 3[x+1]$ or

$3x + 3$ 1

f) $\int \frac{3x}{x^2+1}\, dx$

1 $= \frac{3}{2} \int \frac{2x}{x^2+1}\, dx$

$= \frac{3}{2} \ln(x^2+1) + C$

g) $\sin\theta = \sqrt{3}\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \sqrt{3}$

$\tan\theta = \sqrt{3}$ 1

$\theta = \frac{\pi}{3}, \frac{4\pi}{3}$ 1

Q12

a) I) (2) $y = \frac{3x}{7\cos x}$

$y' = \frac{3}{7}\left[\frac{\cos x \times 1 + x\sin x}{\cos^2 x}\right]$

$= \frac{3}{7}\left[\frac{\cos x + \sin x}{\cos^2 x}\right]$

II) (2) $y = 4x^2 \ln(2-x)$

$y' = \frac{-4x^2}{2-x} + 8x\ln(2+x)$

$u = 4x^2$

$u' = 8x$

$v = \ln(2-x)$

$v' = \frac{-1}{2-x}$

b) i) (2) $A(2,-2)$, $C(0,2)$

the gradient of AC $= \frac{2+2}{0-2}$

$= -2$

The equation of AC is

$y - 2 = -2(x - 0)$

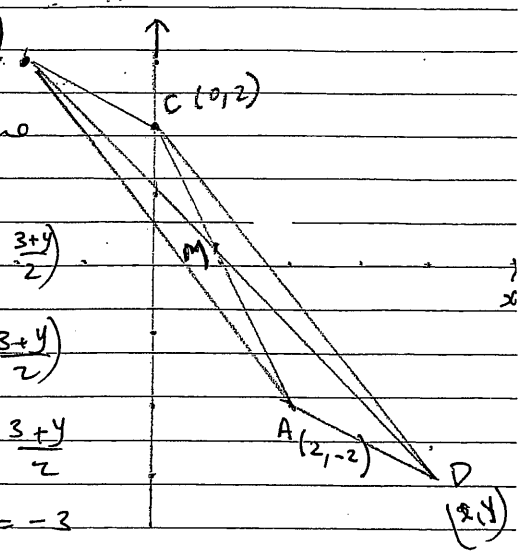
$y - 2 = -2x$

$\therefore 2x + y - 2 = 0$

ii) (2) $d = \frac{|2x - 2 + 3 - 2|}{\sqrt{2^2+1^2}}$

$= \frac{3}{\sqrt{5}}$ or $\frac{3\sqrt{5}}{5}$

iii)



Midpoint of AC is the same as Midpoint of BD

$\therefore \left(\frac{0+2}{2}, \frac{2-2}{2}\right) = \left(\frac{-2+x}{2}, \frac{3+y}{2}\right)$

$(1, 0) = \left(\frac{-2+x}{2}, \frac{3+y}{2}\right)$

$1 = \frac{-2+x}{2}$ $0 = \frac{3+y}{2}$

$\therefore x = 4$ $y = -3$

$D(4, -3)$

c) $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$

$= \frac{\cos\theta}{1-\tan\theta} \times \frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{1-\cot\theta} \times \frac{\sin\theta}{\sin\theta}$

$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} + \frac{\sin^2\theta}{\sin\theta - \cos\theta}$

$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta}$

$= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)}$

$= \cos\theta + \sin\theta$

d) $y = \ln\left(\frac{2x-3}{x^2-6}\right)$

$y = \ln(2x-3) - \ln(x^2+6)$

$y' = \frac{2}{2x-3} - \frac{2x}{x^2+6}$

1. ✓ 5. ✓
2. ✓ 6. ✓
3. ✓ 7. ✓
4. ✓

Question 13

a) $f(x) = 1 - 3x + x^3 \qquad -2 \le x \le 3$

i) $f'(x) = -3 + 3x^2$

For stationary pt $f'(x) = 0$

$-3 + 3x^2 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

at $x = 1$ $\quad f(1) = 1 - 3(1) + 1^3 = 1 - 3 + 1 = -1$

$\therefore P_1(1, -1)$

at $x = -1$ $\quad f(-1) = 1 + 3 - 1 = 3$

$\therefore P_2(-1, 3)$

$f''(x) = 6x$

at $x = 1$ $\quad f''(1) = 6 > 0$ $\therefore$ Minimum Stationary Point

at $x = -1$ $\quad f''(-1) = -6 < 0$ $\therefore$ Maximum Stationary Point

$\therefore$ $P_1(1, -1)$ Min Value

$P_2(-1, 3)$ Max Value

(3)

ii) Point of inflexion $f''(x) = 0$

$6x = 0$

$x = 0$

x	0^-	0	0^+
$f''(x)$	$-$	0	$+$

Since change in concavity

$\therefore$ P.O.I at $x = 0$

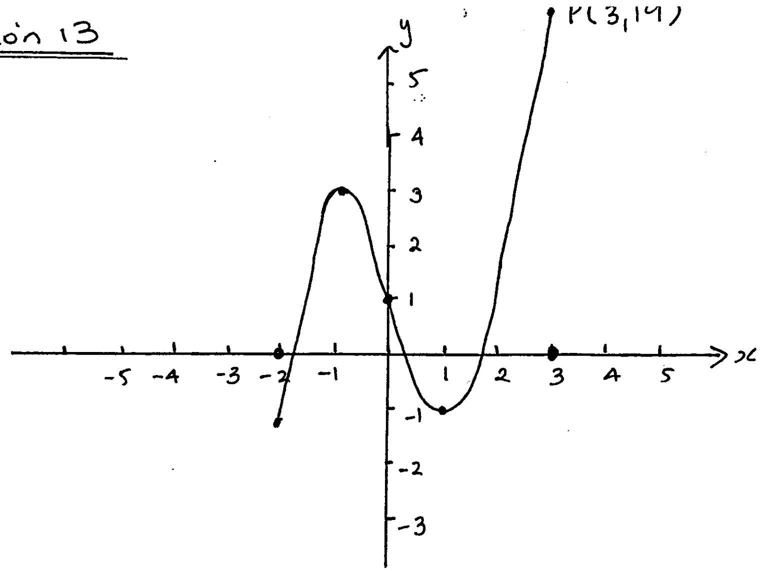
$f(0) = 1$

$\therefore$ P.O.I $(0, 1)$

(2)

Question 13

a) iii)



(2)

$f(x) = 1 - 3x + x^3$

$f(-2) = 1 + 6 + -8 = -1$

$f(3) = 1 - 3(3) + 3^3 = 1 - 9 + 27 = 19$

(1)

iv) Max Value $= 19$

(1)

b) $\int 1 + \sec^2 \pi x \, dx = x + \frac{1}{\pi} \tan \pi x + C$

(1)

c) $y = 3x - p + 2$

$y = x^2 + 1$

$x^2 + 1 = 3x - p + 2$

$x^2 - 3x + p - 1 = 0$

$\Delta = b^2 - 4ac$

$\Delta = 0$

$(-3)^2 - 4(1)(p - 1) = 0$

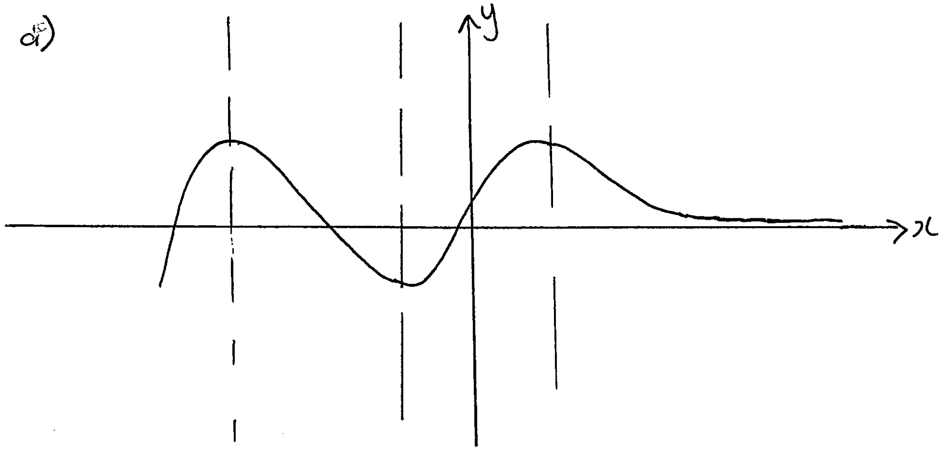
$9 - 4p + 4 = 0$

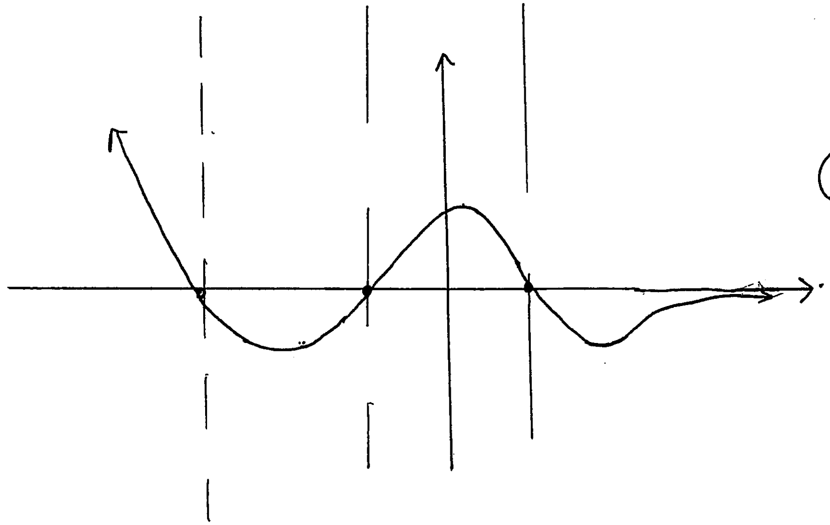
$-4p + 13 = 0$

$\therefore \quad p = \frac{13}{4}$

(2)

13 d)





(2)

e) $5^{2x} - 4.5^{x} - 5 = 0$

let $m = 5^{x}$

$m^2 - 4m - 5 = 0$

$(m-5)(m+1) = 0$

$m = 5$ or $m = -1$

$5^{x} = 5$ $\quad$ $5^{x} = -1$

$x = 1$ $\quad$ No solution

(2)

Question 14

1. ✓ 2. ✓ 3. ✓ 4. ✓ 6. ✓ 7. ✓

a) $y = 3\cos 2x$

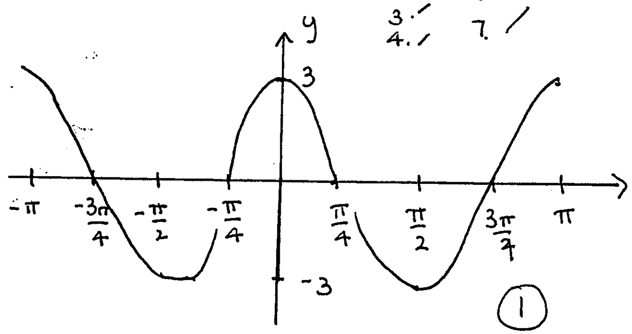
i) Amplitude = 3

Period = $\frac{2\pi}{2}$

$= \pi$

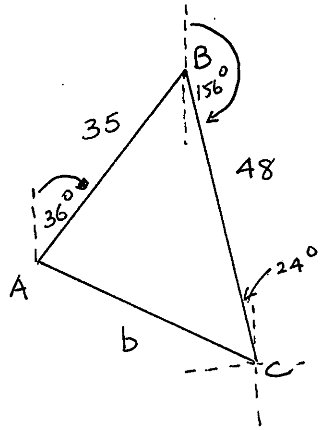
(2)

ii)



(1)

b)



i) $\angle ABC = 36 + (180 - 156)$

$= 36 + 24$

$= 60°$

(2)

ii) $b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = (48)^2 + (35)^2 - 2(48)(35)\cos 60°$

$b^2 = 1849$

$b = \sqrt{1849}$

$b = 43$

(2)

iii) $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \frac{48^2 + 43^2 - 35^2}{2(48)(43)}$

$\cos C = 0.709$

$C = 44°49'$

$\therefore$ Bearing of A from C $= 360° - 44°49' - 24$

$= 291°11'$

(2)

14 d) $2y = x^2 - 4x + 6$

$2y = (x-2)^2 + 2$

$2(y-1) = (x-2)^2$

General Eqn,

$(x-b)^2 = 4a(y-c)$

$\therefore$ i) Vertex (b, c)

$(2, 1)$ (2)

ii) Focus

Focal length $= a$

$\therefore 4a = 2$

$a = \frac{1}{2}$ (1)

$\therefore$ Focus $(2, 1\frac{1}{2})$

iii) Directrix $y = \frac{1}{2}$ (1)

e) $y = 2xe^x$

let $u = 2x$, $v = e^x$

$\frac{du}{dx} = 2$ $\quad \frac{dv}{dx} = e^x$

$\therefore \frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$= 2e^x + 2xe^x$

$\frac{dy}{dx} = 2e^x(1+x)$

at $x = 1$

$\frac{dy}{dx} = 2e(2)$

$m = 4e$

$\therefore$ Eqn of tangent

$y - y_1 = m(x - x_1)$

$y - e = 4e(x-1)$

$y - e = 4ex - 4e$

$y = 4ex - 3e$

or

$4ex - y - 3e = 0$

(2)

15) $\ln(2x+2)x - \log_e 12 = 0$

$\ln(2x^2+2x) = \log_e 12$

$2x^2 + 2x - 12 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3$ or $x = +2$

$x \neq -3$

$\therefore x = 2$

b) In Δs SAB & SUT

$\angle S$ is common

$\frac{SB}{ST} = \frac{6}{9} = \frac{2}{3}$

$\frac{SA}{SU} = \frac{8}{12} = \frac{2}{3}$

$\therefore \Delta SAB \,|||\, \Delta SUT$ (equiangular)

$\frac{AB}{15} = \frac{2}{3}$

$AB = \frac{30}{3}$

$= 10$

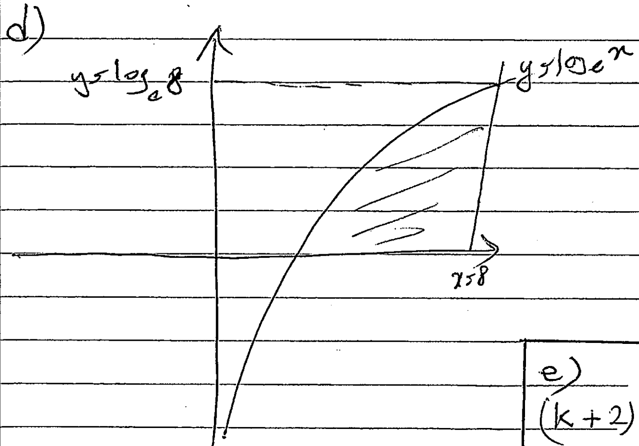
c)

x	$f(x)$	w	wf(x)
0	10	1	10
7	27	4	108
14	19	2	38
21	8	4	32
28	0	1	0

$A \doteqdot \frac{7}{3}(188)$

$\doteqdot 438\frac{2}{3}\ u^2$

d)



$A = 8 \times \ln 8 - \int_0^{\ln 8} e^y\, dy$

$= 8\ln 8 - \left[e^y\right]_0^{\ln 8}$

$= 8\ln 8 - \left[e^{\ln 8} - 1\right]$

$= 8\ln 8 - 8 + 1$

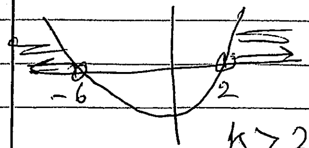
$= 8\ln 8 - 7\ u^2$

e) $(k+2)^2 - 16 > 0$

$k^2 + 4k + 4 - 16 > 0$

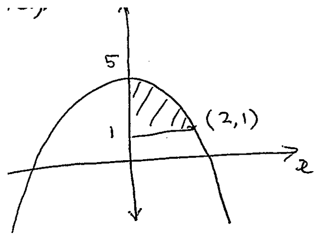
$k^2 + 4k - 12 > 0$

$(k+6)(k-2) > 0$



$k > 2$

$k < -6$



$y = 5 - x^2$

$V_1 = \pi \int_0^2 (5 - x^2)^2 \, dx$

$= \pi \int_0^2 (25 - 10x^2 + x^4) \, dx$

$= \pi \left[25x - \frac{10x^3}{3} + \frac{x^5}{5} \right]_0^2$

$= \pi \left(25 \times 2 - \frac{10 \times 2^3}{3} + \frac{2^5}{5} \right) - 0$

$= \frac{446\pi}{15} u^3$

$V_2 = \pi \times 1^2 \times 2$

$= 2\pi u^3$

$\therefore$ Reqd. vol $= \frac{446\pi}{15} - 2\pi$

$= \frac{416\pi}{5} u^3$

Area $= \frac{1}{2} r^2 \theta$

$8 = \frac{1}{2} r^2 \theta$

$r^2 \theta = 16$

$r^2 = \frac{16}{\theta}$

$r = \frac{4}{\sqrt{\theta}}$ (as $r > 0$)

ii) $l_{BC} = r\theta$

$= \frac{4}{\sqrt{\theta}} \times \theta$

$= 4\sqrt{\theta}$ cm

$P = 2r + l_{BC}$

$= 2 \times \frac{4}{\sqrt{\theta}} + 4\sqrt{\theta}$

$= \frac{8}{\sqrt{\theta}} + 4\sqrt{\theta}$

iii) $P = 8\theta^{-\frac{1}{2}} + 4\theta^{\frac{1}{2}}$

$P' = -4\theta^{-\frac{3}{2}} + 2\theta^{-\frac{1}{2}}$

$= -\frac{4}{\theta\sqrt{\theta}} + \frac{2}{\sqrt{\theta}}$

For min perimeter, $P' = 0$ and $P'' > 0$

$-\frac{4}{\theta\sqrt{\theta}} + \frac{2}{\sqrt{\theta}} = 0$

$\frac{-4 + 2\theta}{\theta\sqrt{\theta}} = 0$

$2\theta = 4$

$\theta = 2$ radians

$P'' = 6\theta^{-\frac{5}{2}} - \theta^{-\frac{3}{2}}$

$= \frac{6}{\theta^2\sqrt{\theta}} - \frac{1}{\theta\sqrt{\theta}}$

when $\theta = 2$

$P'' > 0$

$\therefore$ min. perimeter when $\theta = 2$ radians

c) ii) $A_1 = M \times 1.005 - 1800$

$A_2 = (A_1 + M)1.005 - 1800$

$= ([M \times 1.005 - 1800] + M)1.005 - 1800$

$= M \times 1.005^2 - 1800 \times 1.005 + 1.005M - 1800$

$= (1.005^2 + 1.005)M - 1800(1.005 + 1)$

$A_3 = (A_2 + M)1.005 - 1800$

$= ([M \times 1.005^2 - 1800 \times 1.005 + 1.005M - 1800])1.005 - 1800$

$= M \times 1.005^3 - 1800 \times 1.005^2 + 1.005^2 M - 1800 \times 1.005 - 1800$

$= (1.005^3 + 1.005^2)M - 1800(1.005^2 + 1.005 + 1)$

$A_n = (1.005^n + 1.005^{n-1} + \ldots + 1.005)M - 1800(1 + 1.005 + 1.005^2 + \ldots + 1.005^{n-1})$

$A_{60} = (1.005 + 1.005^2 + \ldots + 1.005^{60})M - 1800(1 + 1.005 + \ldots + 1.005^{59})$

but $A_{60} = 80000$

$80000 = (1.005 + 1.005^2 + \ldots + 1.005^{60})M - 1800(1 + 1.005 + \ldots + 1.005^{59})$

$= \left[\frac{1.005(1.005^{60} - 1)}{1.005 - 1}\right]M - 1800\left[\frac{1(1.005^{60} - 1)}{1.005 - 1}\right]$

$M\left[\frac{1.005(1.005^{60} - 1)}{0.005}\right] = 80000 + 1800\left[\frac{1.005^{60} - 1}{0.005}\right]$

$M = \$2931.96$ (nearest cent)

iii) $2931.96\left[\frac{1.005(1.005^n - 1)}{0.005}\right] = 120000 + 1800\left[\frac{1.005^n - 1}{0.005}\right]$

$2931.96\left[201(1.005^n - 1)\right] = 120000 + 360000\left[1.005^n - 1\right]$

$2931.96\left[201(1.005^n - 1)\right] - 360000\left[1.005^n - 1\right] = 120000$

$(1.005^n - 1)\left[2931.96 \times 201 - 360000\right] = 120000$

$1.005^n - 1 = \frac{120000}{2931.96 \times 201 - 360000}$

$1.005^n = \frac{120000}{2931.96 \times 201 - 360000} + 1$

$n = 84.38$ months

$\therefore$ 7 years 1 month